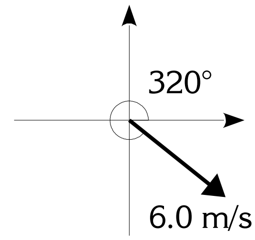
Physics Reference: Vectors

A **vector** is something that has both an amount (called a **magnitude**) and a direction. For example, forces have an amount or strength that we measure in Newtons, but knowing the strength of a force isn't enough: the direction of the force is also important. In one-dimensional problems, + and − signs are enough to show the direction of a vector, but in two dimensions you need angles.



Vector notation

Writing and reading about vectors can be confusing because sometimes you want to talk about the entire vector (magnitude and direction), sometimes you want to talk about just the magnitude, and sometimes you want to talk about the components. Here is a list of the standard styles:

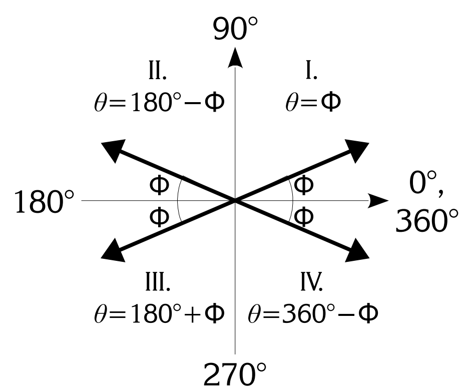
Notation	Description	Example
$\vec{\mathbf{A}}$ or $\vec{\mathbf{A}}$	a vector named "A"	[see drawing above]
A or $\lvert\vec{\mathbf{A}}\rvert$	the magnitude of the vector $\vec{\mathbf{A}}$	A=6.0 m/s
A_x, A_y	x and y components of $\vec{\mathbf{A}}$	A_x=4.6 m/s, A_y=−3.9 m/s
$\vec{\mathbf{A}}=(A_x, A_y)$	both components written as an ordered pair	$\vec{\mathbf{A}}$=(4.6 m/s, −3.9 m/s)
θ, θ_A, or A_θ	the direction of $\vec{\mathbf{A}}$	θ=320°

As you might guess from the "absolute value" notation for a magnitude, the magnitude of a vector is never negative, although the components can be. In a drawing or diagram of vectors, a number written next to the arrow represents the magnitude.

Angle measures

The standard way of stating a vector's direction is with an angle named θ ("theta"). The angle θ must be measured starting at the positive x-axis, rotating counterclockwise. An example can be seen at the top of the page. (This is just like **polar coordinates**, if you've seen them in math!)

Another way of measuring vector direction is with a **reference angle** called Φ ("phi"). The reference angle Φ is measured from the x-axis to the vector by the shortest route. Φ is sometimes more convenient than θ, but has a disadvantage: if all you know is Φ, there are still four different directions that the vector might point. For example, all four vectors at the right have Φ=20°.



Depending on which quadrant you're in, you'll need to use formulas to convert between Φ and θ.

If you want to state a vector's direction in a way other than Φ or θ, you should draw a diagram or say clearly how you are measuring, like "20° south of west" or "30° from the vertical".

Decomposing vectors (getting components)

Given a vector's magnitude and direction in terms of θ, you can get the components this way:

$$A_x = A \cdot \cos(\theta) \qquad A_y = A \cdot \sin(\theta)$$

Components can come out either positive or negative, depending on the direction of $\vec{\mathbf{A}}$. For example, a vector in Quadrant II points left and up, so it will have a negative A_x and a positive A_y.

Breaking a vector into its components is often called **decomposing** or **resolving**.

Recomposing vectors (building from components)

Given the components of a vector, you can find the vector's magnitude and direction. To "build" a vector from its components, use these formulae:

$$A = \sqrt{A_x^2 + A_y^2} \qquad \phi = \tan^{-1}\left|\frac{A_y}{A_x}\right| \qquad \theta = [\text{depends on Quadrant}]$$

To get θ, use the components to determine its quadrant, then calculate it based on Φ.

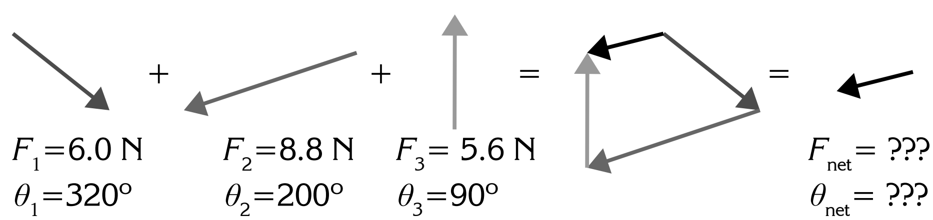
Note: Although A_x and A_y can be negative, the negative signs have no effect on A and Φ. One formula squares the components, and the other includes absolute value bars.

Adding vectors

To combine the effects of two or more vectors into a single **resultant** vector, you must add them. Visually, this means to "follow" each vector in turn, one after another, as shown in the picture. Mathematically, it means to follow these steps:

1. Decompose all the vectors into their components.
2. Add all the x-components together to find the resultant x component. Do the same for y.
3. Recompose the resultant x and y components into the resultant vector.

In the worked example below, three forces ($\vec{\mathbf{F}}_1$, $\vec{\mathbf{F}}_2$, and $\vec{\mathbf{F}}_3$) are added to find $\vec{\mathbf{F}}_{net}$.



F_1:	F_{1x} =6.0·cos(320°)=+4.6 N	F_{1y} =6.0·sin(320°)=−3.9 N
F_2:	F_{2x} =8.8·cos(200°)=−8.3 N	F_{2y} =8.8·sin(200°)=−3.0 N
F_3:	F_{3x} =5.6·cos(90°) = 0.0 N	F_{3y} =5.6·sin(90°) =+5.6 N
F_{total}:	$F_{net,x}$=+4.6−8.3+0.0=<u>−3.7 N</u>	$F_{net,y}$=−3.9−3.0+5.6=<u>−1.3 N</u>

From these components we can tell that the resultant vector is in the third quadrant (both x and y are negative). This means that θ will be 180°+Φ. Therefore:

$$F_{net} = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.7)^2 + (-1.3)^2} = \underline{3.9\ \text{N}}$$
$$\phi = \tan^{-1}|A_y / A_x| = \tan^{-1}|-1.3/-3.7| = 19°$$
$$\text{QIII: } \theta = 180° + 19° = \underline{199°}$$

$\rightarrow \quad \therefore \underline{F_{net} = 3.9\ \text{N at } 199°}$