Reference: Square Roots and Complex Numbers

Square roots

- Every non-negative real number, x, has two square roots: $\sqrt{x}$ and $-\sqrt{x}$. The positive root is also called the **principal square root**. The "square root symbol" is called a **radical**.
- If you square root an integer, the result will either be an integer (if you started with a perfect square) or an irrational number.
  - *Do not try to write irrational numbers as decimals!!* Write " $\sqrt{2}$ ", not "1.4142..."
- Square roots have these properties, assuming a and b are positive real numbers (or zero):
  - $\sqrt{ab}=\sqrt{a}\sqrt{b}$ For example, $\sqrt{80}=\sqrt{10}\sqrt{8}=\sqrt{4}\sqrt{20}=\sqrt{16}\sqrt{5}$ and so on.
  - $\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}$ For example, $\sqrt{\frac{5}{9}}=\frac{\sqrt{5}}{\sqrt{9}}=\frac{\sqrt{5}}{3}$.
  - $\sqrt{a}\sqrt{a}=a$ For example, $\sqrt{6}\sqrt{6}=\sqrt{36}=6$.
  - $\sqrt{x^2}=|x|$ where x is *any* real number, including negatives.
  - Note that this is *not* allowed: $\sqrt{a+b}\neq\sqrt{a}+\sqrt{b}$.
- In the set of real numbers, $\mathbb{R}$, square roots of negative numbers are undefined / don't exist.
- To **fully simplify** an expression with square roots, factor all perfect squares out from the inside of radicals and evaluate them:
  - $\sqrt{180} \rightarrow \sqrt{9}\sqrt{20} \rightarrow \sqrt{9}\sqrt{4}\sqrt{5} \rightarrow 3\cdot 2\sqrt{5} \rightarrow 6\sqrt{5}$
  - ***Honors students***: You *also* need to **rationalize** the denominator if you see a radical down there! Multiply the top and bottom by that value so the bottom becomes rational.
    $\frac{2}{\sqrt{5}} \rightarrow \frac{2}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} \rightarrow \frac{2\sqrt{5}}{5}$

Perfect squares

- A number you get by squaring an integer is called a **perfect square**.
- A quadratic expression you get by squaring a single factor is called a **perfect square quadratic**. It's easiest to recognize a perfect square quadratic if a and c are perfect squares. But, whether they are or aren't, a perfect square quadratic must have $b=\pm 2\sqrt{a}\sqrt{c}$.
- Or, if you'd rather look at c, a perfect square quadratic must have $c=b^2/(4a)$.

Imaginary and complex numbers

- The set of **imaginary numbers** is based on this definition: $i=\sqrt{-1}$, or $i^2=-1$.
- To take a square root of a negative number, treat it EXACTLY like you would a positive square root, including simplifications, and then put an "i" on the end:
  - $\sqrt{-36}=6i$, $\sqrt{-20}=2\sqrt{5}\,i$, etc.
- The set of complex numbers, $\mathbb{C}$, contains numbers of this form: $a+bi$, where a is called the **real part** and b is called the **imaginary part**.
- When adding/subtracting complex numbers, don't combine real with imaginary parts. Keep them separate:
  - $3+2i \;+\; 7-8i \;=\; 10-6i$
- When multiplying complex numbers, "F.O.I.L." them like anything else, but don't forget to change the i^2 into a -1! This changes the sign on that term and gets rid of the i^2.
  - $(3+2i)(7-8i) \rightarrow 21-24i+14i-16i^2 \rightarrow 21-24i+14i+16 \rightarrow 37-10i$
- **Complex conjugates** are pairs of complex numbers like $2+4i$ and $2-4i$, or $10-6i$ and $10+6i$. When complex conjugates are multiplied, the result is a pure real number with no imaginary part:
  $(10+6i)(10-6i) \rightarrow 100-60i+60i-36i^2 \rightarrow 100-60i+60i+36 \rightarrow 100+36 \rightarrow 136$.
- To divide by a complex number, first find its complex conjugate. Then, multiply the top AND bottom of the fraction by that conjugate. Simplify it, and break the answer into real and imaginary parts with the $a+bi$ form. An example:
  $\frac{5-2i}{3+4i} \rightarrow \frac{5-2i}{3+4i}\left(\frac{3-4i}{3-4i}\right) \rightarrow \frac{15-20i-6i+8i^2}{9-12i+12i-16i^2} \rightarrow \frac{15-20i-6i-8}{9-12i+12i+16} \rightarrow \frac{7-26i}{9+16} \rightarrow \frac{7-26i}{25} \rightarrow \frac{7}{25}+\frac{26}{25}i$