Algebra 2 Reference: Radicals & Powers

Properties of powers and roots

- *a* and *b* are nonzero, real numbers. *m* and *n* are integers.

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \qquad a^{-n}=\frac{1}{a^n} \qquad \frac{a^m}{a^n}=a^{m-n} \qquad (a^m)^n=a^{m\cdot n}$$

$$(a\cdot b)^n=a^n\cdot b^n \qquad a^0=1 \qquad a^m\cdot a^n=a^{m+n} \qquad \sqrt[n]{a^m}=a^{\frac{m}{n}}$$

- To simplify an expression like $\sqrt[n]{x^n}$ ... if *n* is ODD, $\sqrt[n]{x^n}=x$. If *n* is EVEN, $\sqrt[n]{x^n}=|x|$.

Solving equations

- Powers are roots are **inverses**, or opposites, of each other. So, you can use them to cancel each other out!
- If your roots and powers are all ODD numbers, there is nothing extra to worry about. But...
  - If you add an EVEN root to an equation, put a ± sign on one side of the equation.
  - If you add an EVEN power to an equation, check your answers for **extraneous solutions**.
  - With a rational exponent, an even numerator (top) is just like an even power, and an even denominator (bottom) is just like an even root. Same rules apply!
- Examples using only ODD roots and powers:
  - $x^5=32 \rightarrow \sqrt[5]{x^5}=\sqrt[5]{32} \rightarrow x=\sqrt[5]{32} \rightarrow x=2$
  - $\sqrt[3]{x}=4 \rightarrow (\sqrt[3]{x})^3=4^3 \rightarrow x=4^3 \rightarrow x=64$
  - $x^{\frac{5}{3}}=32 \rightarrow (x^{\frac{5}{3}})^{\frac{3}{5}}=32^{\frac{3}{5}} \rightarrow x^1=32^{\frac{3}{5}} \rightarrow x=\sqrt[5]{32^3}=(\sqrt[5]{32})^3=2^3=8$
- Examples using EVEN roots and powers:
  - $x^4=16 \rightarrow \sqrt[4]{x^4}=\pm\sqrt[4]{16} \rightarrow x=\pm\sqrt[4]{16} \rightarrow x=\pm 2$
  - $\sqrt[4]{x}=-3 \rightarrow (\sqrt[4]{x})^4=(-3)^4 \rightarrow x=(-3)^4 \rightarrow x=81$
    Check: $\sqrt[4]{81}=-3? \rightarrow 3\neq -3 \rightarrow$ Solution is false.
- You should always start by isolating the radical/power that you want to deal with on one side:
  - $6(2x-3)^{4/5}-2=16 \rightarrow 6(2x-3)^{4/5}=18 \rightarrow (2x-3)^{4/5}=3$
- If there's more than one radical/power in the equation, deal with them one at a time. It's easiest if you do the scariest one first.

Example: an equation with a power

Start!	$(7x+2)^2-11=110$
Add 11 to isolate the weird part.	$(7x+2)^2=121$
The inverse of squaring is a square root. And since we're taking an EVEN root, include a ±.	$\sqrt{(7x+2)^2}=\pm\sqrt{121}$
The inverses cancel out. Simplify the right.	$7x+2=\pm 11$
Move the 2, then divide by the 7.	$x=\frac{\pm 11-2}{7}$
Calculate the answers: No need to check the answers, since we didn't use any powers to solve this: just roots.	$x=9/7, -13/7$

Example: an equation with a radical

Start! $\sqrt[3]{x+7}-5=-1$

Add 5 to isolate the weird part. $\sqrt[3]{x+7}=4$

The inverse of a 3rd root is the 3rd power. $\left(\sqrt[3]{x+7}\right)^3=(4)^3$

The inverses cancel out. Simplify the right. $x+7=64$

This is a linear equation, so solving is easy! $x=64-7$

$x=57$

No need to check it because we used an ODD power!

Example: an equation with a rational exponent

Start! $(4-x)^{2/3}+3=12$

Subtract 3 to isolate the weird part. $(4-x)^{2/3}=9$

The inverse of a 2/3 power is 3/2 power. And since we're using an EVEN denominator, include a $\pm$. $\left((4-x)^{2/3}\right)^{3/2}=\pm 9^{3/2}$

The inverses cancel out. Simplify the right. $4-x=\pm 27$

Move the 4 to start isolating x. $-x=\pm 27-4$

Multiply everything by -1 so x is positive. $x=\pm 27+4$

Calculate the answers: $x=+27+4=31$,

OR $x=-27+4=-23$

$x=-23, 31$

We don't need to check the answers because the numerator of the power we used was ODD.

Example: an equation with two radicals!

Start! $\sqrt{2x+1}+\sqrt{x}=1$

Isolate the weirder radical first. $\sqrt{2x+1}=1-\sqrt{x}$

The inverse of a square root is squaring. $\left(\sqrt{2x+1}\right)^2=\left(1-\sqrt{x}\right)^2$

The inverses cancel out. Simplify the right. $2x+1=1-2\sqrt{x}+x$

Simplify a little by combining like terms... $x+1=1-2\sqrt{x}$

$x=-2\sqrt{x}$

The radical term is isolated, so square again. $x^2=\left(-2\sqrt{x}\right)^2$

Simplify. $x^2=4x$

This is a quadratic equation now. Get it in standard form. $x^2-4x=0$

Factor. $x(x-4)=0$

Each factor gives a solution. $x=0, 4$

We used an EVEN power to solve, so time to check the solutions. $\sqrt{2\cdot 0+1}+\sqrt{0}=1$

$\sqrt{1}+0=1\rightarrow TRUE$

$\sqrt{2\cdot 4+1}+\sqrt{4}=1$

$\sqrt{9}+\sqrt{4}=1\rightarrow FALSE$

Report only the true solutions: $x=0$