Algebra 2 Reference: Polynomial Expressions

Roots and zeros

Suppose the number "*r*" is a **root** or a **zero** for some polynomial function, $P(x)$. This means...

- $P(r) = 0$. (That is, if you plug in "*r*", it should simplify to zero.)
- $(x-r)$ is a factor of the polynomial.
- If you divide the polynomial by $(x-r)$, you will get a remainder of zero.
- "*r*" is a solution to the equation $P(x) = 0$.
- At $x=r$, the graph crosses $y=0$. (That is, it has an *x*-intercept at *r*.)
- *ALL OF THE ABOVE STATEMENTS ARE EQUIVALENT!*

Factoring

You can (sometimes) use these techniques to partially factor a polynomial!

- Common factor that includes *x*
    - If all terms in the polynomial have at least one factor of *x*... you can factor out some *x*.
      $12x^5+4x^4+20x^2 \rightarrow 4x^2(3x^3+x^2+5)$
- Sum/Difference of Cubes
    - If it's just two terms and they're both cubes, use this pattern: $a^3+b^3=(a+b)(a^2-ab+b^2)$
      $125x^3-27 \rightarrow$ cube roots are 5 and -3 $\rightarrow (5x+(-3))(25x^2-5(-3)x+9)$
      $\rightarrow (5x-3)(25x^2+15x+9)$
- Factor by grouping
    - If you have four terms, try taking a common factor from your first two terms, and another from the next two terms. If you get the same "leftovers" both times... you can factor it.

Working example:	Non-working example:
x^3+2x^2-3x-6	x^3+2x^2-3x-9
$x^2(x+2)-3(x+2)$	$x^2(x+2)-3(x-3)$
Same thing both times!	Different things.
$(x+2)(x^2-3)$	Trick won't work. ;_;

- Substitution
    - If all the exponents on your "*x*" terms are multiples of 2, or 3, or whatever, you can make up a new variable, like $a=x^{whatever}$. This makes the problem easier to look at, especially if it means the polynomial now looks like a quadratic!
    - Just remember to put back the original variable later!
      $x^{10}-5x^5-14 \rightarrow$ Let $a=x^5$. $\rightarrow a^2-5a-14 \rightarrow (a+2)(a-7) \rightarrow (x^5+2)(x^5-7)$

How to find all zeros/roots/solutions for a polynomial

- Make sure it's written in standard form!!
- If you can use any factoring “tricks” to lower the degree, try that first. If not...
- Make a list of the possible rational zeros of your polynomial, like “m/n”:
  - the numerator, m, must be a factor of the <u>constant term</u>
  - the denominator, n, must be a factor of the <u>leading coefficient</u>
- Check these possible zeros, one by one, by plugging them in to see if you get zero!
- When you find one that works, write its corresponding factor. Factor it out by doing a long division problem.
- Repeat this process until completely factored! A polynomial's degree is the same as its number of zeros, if you include the irrational and complex roots (with “i”).

Long division example

- Long division of polynomials is very similar to long division of numbers! There's a four-step cycle that you go through over and over, until the problem is complete:
  1. DIVIDE the leading terms of both polynomials. You don't have to worry about the other terms during this step. Write the answer on top.
  2. MULTIPLY the answer you just wrote by the entire divisor. In the example, this means multiply by “$4x$-3”. Write the answer underneath.
  3. SUBTRACT the result you just wrote from the terms above it. I recommend putting parentheses around it to help you get the minus signs straight.
  4. BRING DOWN the next term from of the original polynomial.
- You can tell you're “done” when the result of your subtraction is of a lower degree than your divisor. In the example, I have a linear (degree 1) divisor, so I stop when my subtraction gives me a constant (degree 0) answer.
- If your polynomials “skip” any terms, you should include a “0” term when you set up the problem. For example, instead of “$5x^4+2$”, write “$5x^4+0x^3+0x^2+0x+2$”. If you don't, you'll end up “bringing down” the wrong term at the wrong time.

Set up the problem, including 0 terms for anything that was missing in the original polynomial.

$4x$	-3	$8x^3$	$-6x^2$	$-36x$	30

(1) First time through the cycle:

		$2x^2$			
$4x$	-3	$8x^3$	$-6x^2$	$-36x$	30
		$-(8x^3$	$-6x^2)$		
			$0x^2$	$-36x$	

(2) Second time through:

		$2x^2$	$+0x$		
$4x$	-3	$8x^3$	$-6x^2$	$-36x$	30
		$-(8x^3$	$-6x^2)$		
			$0x^2$	$-36x$	
			$-(0x^2$	$+0x)$	
				$-36x$	$+30$

(3) Third time through:

		$2x^2$	$+0x$	-9	
$4x$	-3	$8x^3$	$-6x^2$	$-36x$	30
		$-(8x^3$	$-6x^2)$		
			$0x^2$	$-36x$	
			$-(0x^2$	$+0x)$	
				$-36x$	$+30$
				$-(-36x$	$+27)$
					3

(4) Answer:

$2x^2$ $+0x$ -9 with remainder 3.