Reference: Factoring Quadratic Expressions

Before trying to factor, make sure the expression is written in standard form: $a x^2 + b x + c$!!!

- First, take out all common factors, including variables. Sometimes, this is all you can do!
  - Example: $5x^2 - 45x \rightarrow 5x(x-9)$

- Then, if the expression is **<u>monic</u>** ($a=1$), find numbers which multiply to make c and add to make b. These numbers go into the factors. Be careful about + and − signs!
  - Example: $x^2 + 7x - 30 \rightarrow$ 10 and -3 $\rightarrow (x+10)(x-3)$

- If not monic ($a \neq 1$), check for a "**<u>perfect square</u>**" or "**<u>difference of two squares</u>**" pattern. For both of these, the values of a and c are perfect squares. (Don't forget "1" is a perfect square!)
  - <u>Difference of two squares</u>: a and c are squares, $b=0$, and there's a minus sign.
    Example: $9x^2 - 16 \rightarrow (3x+4)(3x-4)$
  - <u>Perfect square</u>: a and c are squares, and the b value matches $b = \pm 2\sqrt{a}\sqrt{c}$.
    Example: $9x^2 - 24x + 16 \rightarrow 2 \cdot 3 \cdot 4 = 24$, so b matches $\rightarrow (3x-4)^2$
    Example: $9x^2 + 24x + 16 \rightarrow 2 \cdot 3 \cdot 4 = 24$, so b matches $\rightarrow (3x+4)^2$
    NOTE: if the b value doesn't match $\pm 2\sqrt{a}\sqrt{c}$, it's NOT a perfect square!

- If it doesn't match those patterns, use **<u>Factoring by Grouping</u>** (the "**<u>AC method</u>**"):
  - Calculate $a \cdot c$.
  - Find two numbers which multiply to make $a \cdot c$ but add to make b.
  - Split your b term in two using those numbers.
  - Look at your first two terms and factor out their GCF. (Including an x.)
  - Look at your other two terms and factor out their GCF. (There's no common x this time.)
  - You should get the same stuff in parentheses both times. Pull it out to the front.
  - Example:
    $6x^2 + x - 15 \rightarrow a \cdot c = -90, b = 1 \rightarrow$ so $b_1 = 10$ and $b_2 = -9 \ldots$
    $6x^2 + 10x - 9x - 15 \rightarrow 2x(3x+5) - 3(3x+5) \rightarrow (3x+5)(2x-3)$

- ***Note:*** If you can't find numbers for the AC method that work, then the expression <u>can't be factored further</u>. (It's **<u>prime</u>**, just like a number that can't be factored.) The AC method will work for <u>any</u> factorable quadratic expression, so if you want to use it every time, go for it. It's often slower than the other methods, but it works.

- You *can* use the **<u>Quadratic Formula</u>** to figure out the factors... $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  - Use the formula to find the zeros (or roots) of the expression.
  - If we call the zeros z_1 and z_2, then the factored form is: $a(x - z_1)(x - z_2)$
  - Note that we had to place "a" out in front like a common factor!!
  - Example: $6x^2 + x - 15 \rightarrow$ use formula $\rightarrow -\frac{5}{3}$ and $\frac{3}{2} \rightarrow 6(x + \frac{5}{3})(x - \frac{3}{2})$
  - This was the same example used for the AC method. The factored form comes out looking a little different, but they're equal to each other. Either form is OK!